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An Analysis of Transducer Mass Loading Effect Inshaker Testing

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Abstract
Modal Analysis has been a developing science in the experimental evaluation of the dynamic properties of the structures. Frequency Response Function (FRF) is one of the major steps in modal analysis. Measured frequency response functions (FRFs) are used to extract modal parameters.
It is also known that the accuracy and the reliability of various analyses using the measured FRFs depend strongly on the quality of measured data. It is well known that the quality of measured frequency response functions (FRFs) is adversely affected by many factors, most significant sources being noise and systematic errors like mass loading effects of transducers. A transducer mounted on a vibrating system changes the dynamics of the structure due to the addition of extra mass and introduces errors into measured FRFs. One problem with this is the production of unrealistic results, which cause the measured resonant frequencies to be less than the correct values. These errors also lead to incorrect prediction of modal parameters.
In many situations, the mass loading effect is ignored in the analytical and experimental process, based on a usual assumption that the transducer mass is negligible compared to that of the structure under test. However, when light-weighted structures are investigated, this effect can be significant.
This paper focuses on the theoretical analysis of transverse vibration of fixed free beam and investigates the modal frequency. Mass loading effect of accelerometer is studied on the cantilever beams by varying the masses of accelerometer.
In this work, experimental modal testing of a cantilever beam has been performed to obtain modal frequencies. The beam is excited by using Electrodynamics Shaker Excitation Technique, which provides forced vibrations. These modal parameters are then checked using finite element method which is found to comply with the experimental results. The range of applications for modal data is vast and includes checking modal frequencies, to understand dynamic structural behavior for trouble-shooting, verifying and improving analytical models.
Keywords- Frequency Response Function, Modal Analysis, modal parameters, mass loading effects, Shaker Testing etc.

I. INTRODUCTION

This Frequency response functions (FRFs) measurement is an importance process in modal testing. The quality of FRFs measured on a structure has been a concern of vibrationengineers for a considerable period of time. Accurate FRFs measurement is the prerequisite to obtain high-precision modal parameters. However, the measured FRFs are often inaccurate due to various factors in the testing process. Among these, one of the unavoidable error sources is the so-called mass loading effects of transducers.

In modal testing, some sensors (such as force transducer and accelerometer) have to be mounted on the test structure. The dynamics of the test structure are therefore changed and the measured FRFs contain errors consequently, such as deviation of the measured resonant frequencies from their correct values. It is desirable in practice that these deviations are acceptably small as they may cause considerable difficulties in many applications depending on the level of errors induced by transducer mass loading during measurement.

For large structure under test, the mass loading effects are ignored based on a usual assumption that the transducer mass is negligible compared to that of the structure. However, as the mass of the transducer approaches that of the test article i.e. the test structure is small and lightweight, this effect can be significant. Lightweight structures are those structures that optimize the load carrying capacity of the elements by large deflection, allowing the load to be taken primarily in tension. It is characterized by having small mass relative to the applied load which the shape of the structure is determined through an optimization process. Lightweight structures include cable, membrane, shell, thin plate and folded structures. In such cases, it is necessary to eliminate this undesirable side effect before the measured data are used for further analysis.

II. FREQUENCY RESPONSE FUNCTION

These functions are used in vibration analysis and modal testing. There are many tools available for performing vibration analysis and testing. The frequency response function is a particular tool. A frequency response function (FRF) is a transfer function, expressed in the frequency domain. Frequency response functions are complex functions, with real and imaginary components. They may also be represented in terms of magnitude and phase. A frequency response function can be formed from either measured data or analytical functions.

Consider a linear system as represented by the diagram in Fig.1. F(ω) is the input force as a function of the angular frequency w . H(ω) is the transfer function. X(ω) is the displacement response function. Each function is a complex function, which may also be represented in terms of magnitude and phase.

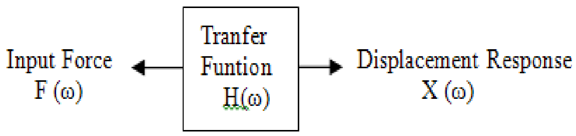

Fig. 1 Frequency Response Function (FRF)

III. THEORY OF MASS LOADING

The mass of an accelerometer can significantly affect the dynamic characteristics of the structure to which it is mounted. This is commonly called mass loading effect which tends to lower the measured natural frequencies. The general rules is the accelerometer mass should be less than one-tenth from the effective mass of the structure to which it is attached. Theoretically, the natural frequency is;

$$\omega = \sqrt{(K/M)}$$

The addition of the accelerometer mass to the mass of the vibrating structure changes the resonant frequency of the vibrating systems as follows;

$$f_m = f_s\sqrt{K/(M+ma)}$$

Where ω = natural frequency
K = stiffness of the structure
M = mass of the structure
Ma = accelerometer mass
f_m = frequency of the structure with the influence of the accelerometer mass
f_s = frequency of the structure without the influence
of the accelerometer mass

This relationship shows that if the accelerometer mass is kept small compared to the mass of the structure then any changes in the vibration will be only small. The mass loading produced by accelerometer depends on the local dynamic properties of the structure. The mass and resulting frequencies shift is proportional to the square of deflection of the associated mode. This study will determine how much the natural frequency will change due to the mass loading effect.

IV. EULER BERNOULLI BEAM THEORY

Euler Bernoulli's Beam Theory also known as engineer's beam theory or classical beam theory is a simplification of the linear theory of elasticity which provides a means of calculating the load carrying and deflection characteristics of beams . It covers the case for small deflections of a beam which is subjected to lateral loads only. It is thus a special case of Timoshenko beam theory which accounts for shear deformation.

The Euler-Bernoulli equation describes the relationship between the beam's deflection and the applied load:

$$\frac{d^2}{dx^2}\left(EI\frac{dw^2}{dx^2}\right) = q$$

Where, E is the elastic modulus
I is the second moment of area
Q is a distributed load (force per length).

I must be calculated with respect to the centroidal axis perpendicular to the applied loading. For an Euler-Bernoulli beam not under any axial loading this axis is called the Neutral axis. Often, EI is a constant, so that:

$$EI\frac{dw^4}{dx^4} = q(x)$$

This equation, describing the deflection of a uniform, static beam, is used widely in engineering practice.

V. FINITE ELEMENT METHOD

Consider the cantilever beam with and without accelero- meter mounted on it. The beam data is as follows:

Dimension	: 302 x 19 x 3 mm
Density	: 7850 kg/m3
Modulus of elasticity	: 210 GPa
Mass of Cantilever Beam	: 148gm
Mass of accelerometer	: 14.4 gm

Using FEM we will find the natural frequencies of the continuous cantilever beam. The Basic procedure is outlined here

1) In the first step, the geometry is divided into a number of small elements. The elements may be of different shapes and sizes.

2) Then elemental equations are obtained for each element.

3) In the third step the elemental equations are assembled to yield a system of global equation.

4) The problem is solved by reduced down to the equation,

$\{[M]\omega 2 + [K]\} X = 0$

The above equation represents the standard Eigen value problem whose solution gives Eigen vectors and Eigen values. The Eigen values represent the square of the natural frequencies and the Eigen vectors represent the corresponding mode shapes.

In MATLAB the cantilever beam is generated with different number of elements until the previous result and current result do have negligible difference. The result is shown in table 4.1 below. From the result it is shown that beyond 10 elements the changes are insignificant.

Therefore for all further simulation 10 element beam is considered

VI. EXPERIMENTAL APPROACH

A.Experimental Modal Analysis

After acquiring all the data by performing measurements at all the DOFs, thus obtaining modal parameters .This part of modal test is called experimental modal analysis as this is the stage of the experimental approach corresponding to the stages called modal analysis also in theoretical approach. In both cases, modal analysis leads to identification of modal properties of the system. However, it should be noticed that these two processes are somehow different: experimental approach deals with the actual measured data, while theoretical analysis deals with the Eigen value problem.

B. Instrument Set up

The instruments used in testing are shown in figure. The experimental vibration consists of following parts:

1. Electro dynamic Vibration Shaker
2. Power Amplifier
3. The Piezoelectric Accelerometer
4. Vibration Controller
5. Interconnecting Cables

C. Measurement Procedure

1) A beam of particular material (steel), dimensions (L,w,d) is used as the cantilever beam.
2) The fixed end is made by clamping the beam on the combo base slip table of electro-dynamic shaker with M8 threaded bolts.
3) The connections of FFT analyzer, laptop, transducers and exciter along with the requisite power connections were made.
4) Placed piezoelectric accelerometer at the free end of the cantilever beam, to measure the forced vibration response (acceleration).
5) Ensured the connection of the exciter with the vibration controller and the level of input power.
6) Connected another accelerometer on the combo slip table of electro-dynamic shaker to measure the input response. Make a proper connection of accelerometer with vibration controller and with computer to capture the vibration data.
7) During setting of the swept-sine parameter make sure that in the vibration measurement software the time duration should be greater than the total time of excitation.
8) Started the experiment by giving force signal to the exciter and allow the beam to forced vibrate.
9) Recorded all the data obtained from the transducer in the form of the vibration response with time.
10) Repeated the experiments to check the repeatability of the experimentation (i.e. vibration data).
11) Recorded the whole set of data in the data base for further processing and analysis.

VII. RESULTS AND DISCUSSION

A. Validation of the FE Model with Euler-Bernoulli Equation

In this value of natural frequencies with and without accelerometer by FEM Method are compared with analytical values. The validation of FE model for steel plate is shown in Table 1. There is negligible difference between values obtained from Euler-Bernoulli Equation and Finite Element Method. The MATLAB Program for FEM Method with and without accelerometer is generated. The output of MATLAB program is shown in Fig. 2 for steel plate.

Table 1: Validation of FE Model for Steel Plate

Natural Frequency (rad/s²)	**Without Accelerometer**		**With Accelerometer**	
	Analytical	**FEM**	**Analytical**	**FEM**
1	31.1473	31.15	26.51	26.52
2	195.1967	195.84	172.5	172.73
3	546.5563	553.37	443.4	500.24

The peak point in graph of Frequency Response Function (FRF) Vs Frequency (ω) corresponds to the natural frequencies. First Peak represents the first natural frequency; second peak represents the second natural frequency and so on.

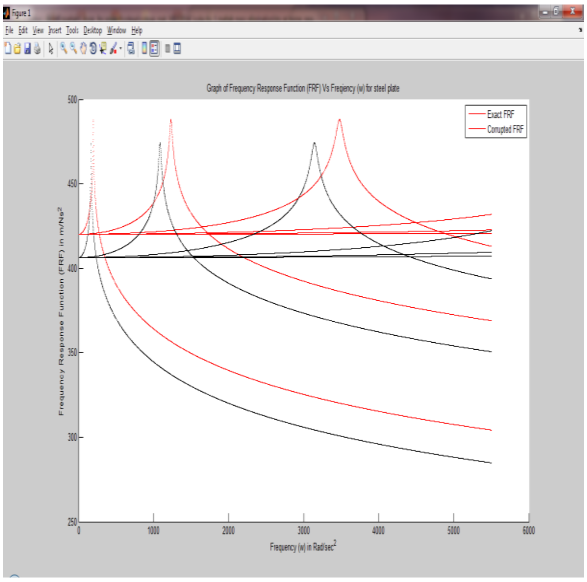

Fig. 2 Graph of Frequency Response Function (FRF) Vs. Frequency (ω) for Steel Plate

B. Percentage increase in mass over Beam Mass

The accelerometer mass is increased in steps of 2 g over mass Of cantilever beam. The mass of steel beam is 148g.This Percentageincrease in mass is shown in Table 2 for steel beam.

Table 2: Percentage increase in mass over beam mass for steel plate

Mass of accelerometer (g)	**Percentage increase of accelerometer mass compared to Steel plate mass (%)**
2.4	1.62
4.4	2.97
6.4	4.32
8.4	5.67
10.4	7.02
12.4	8.37
14.4	10.14

C. The natural frequencies corresponding to various the Accelerometer Masses

The first three natural frequencies are calculated for each accelerometer mass. The first three natural frequencies corresponding to each mass are shown in Table 3 for steel plate.

Table 3: The natural frequencies corresponding to various accelerometer masses for Steel plate

ma (g)	**% of ma**	**1st Mode**	**2nd Mode**	**3rd Mode**
0	0	31.15	195.84	553.37
2.4	1.62	30.22	190.19	538.24
4.4	2.97	29.50	186.22	528.51
6.4	4.32	28.82	182.79	520.64
8.4	5.67	28.19	179.79	514.17
10.4	7.02	27.60	177.16	508.75
12.4	8.37	27.05	174.82	504.17
14.4	10.14	26.52	172.73	500.24

The variation of natural frequencies with accelerometer mass is shown in Fig.3 for steel plate.

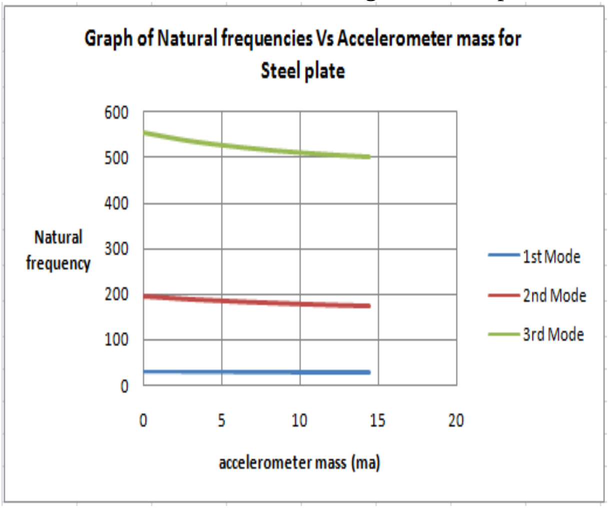

Fig.3 Variation of natural frequencies with accelerometer mass for Steel Plate

D. Accelerometer Mass Loading Error

The accelerometer mass loading error is the percentage variation in natural frequencies with and without accelerometer mass. The accelerometer mass loading error for steel plate is shown in Table 4.

Table 4: Accelerometer mass loading error for Steel Plate

Natural Frequency (rad/s²)	**□1**	**□2**	**□3**
Without accelerometer mass	31.15	195.84	553.37
With accelerometer mass	26.52	172.73	500.24
% error	14.86	11.80	9.60

The mass loading effect of accelerometer on steel beam is shown in Fig. 4.

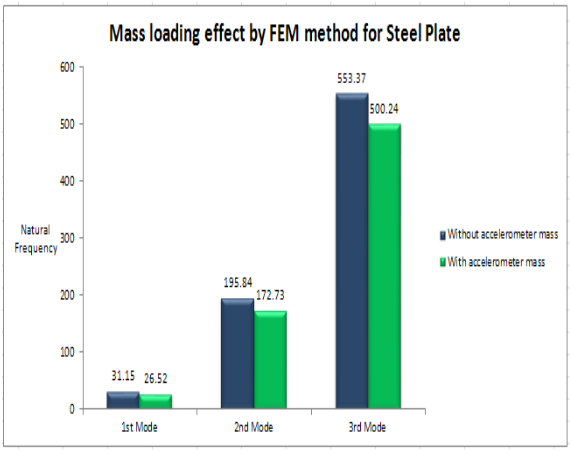

Fig. 4 Accelerometer mass loading effect on Steel Beam

The variation of percentage mass loading error of accelerometer with natural frequency for steel plate is shown in Fig. 5.The percentage error is shown for first three natural frequencies.

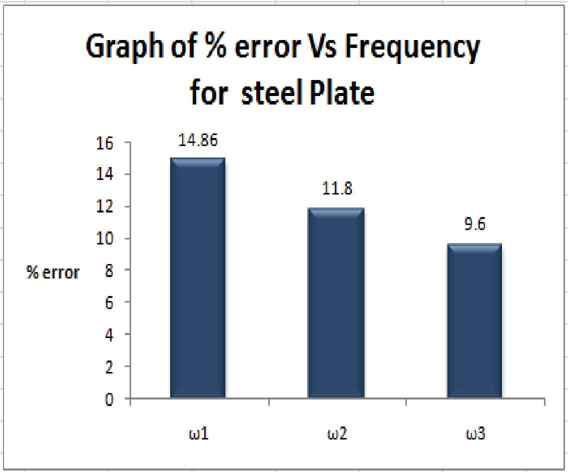

Fig. 5 Variation of percentage mass loading error with frequency for steel plate

E.Results of Shaker Testing

The cantilever beam of both materials is tested through electrodynamics shaker. The graphs of transfer function Vs Frequency are shown in Fig.6 for steel beam.The natural frequencies are corresponding to the peaks in graphs of transfer function Vs frequency.

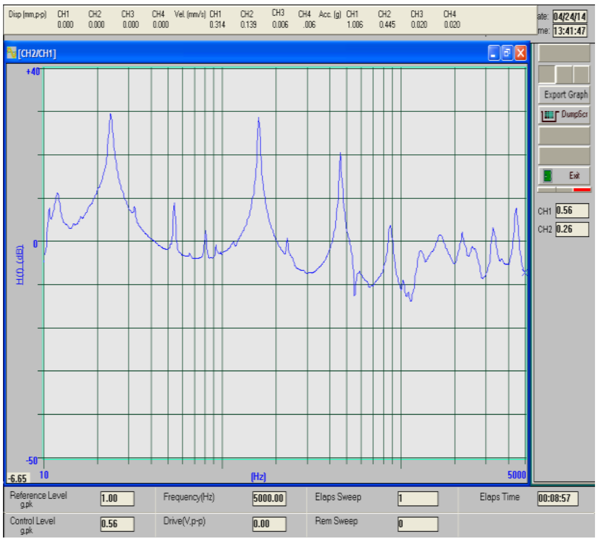

Fig. 6 Variation of transfer function with Frequency for steel beam

F. Validation of Experimental values with FEM

The values obtained from experimental results are compared with those obtained from FEM method. The validation of experimental results with FEM results for steel plate is shown in Table 5.

Table 5: Validation of Experimental results with FEM for steel plate

Sr.No.	Natural Frequency in Hz	
	FEM	Experimental
1	26.52	23.8
2	172.73	160.4
3	500.24	458.8

The comparison of experimental results with FEM results is shown in Fig. 7 for steel plate.

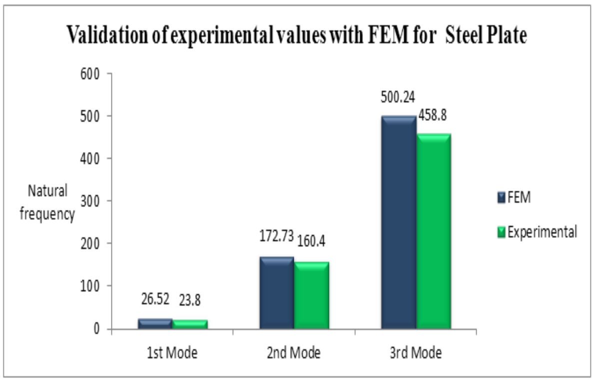

Fig .7 Comparisons of experimental results with FEM for Steel Plate

VIII. CONCLUSION

The vibration analysis of a structure holds a lot of significance in its designing and performance over a period of time. The transverse vibrations of cantilever beam carrying mass at end are investigated using analytical and Finite Element methods. FEM and analytical solution are in good co-ordinance with each other. Since it is tedious and difficult to obtain the frequency equations, FEM will be appropriate to calculate the natural frequencies of beams.

It is seen that the lowest frequency is in 1st mode. The frequency increases with each subsequent mode of vibration. The natural frequencies are calculated for several masses and the comparison of frequencies is presented. It is noted that accelerometer mass should not be more than one-tenth of the mass of the structure. Thus, the lightest accelerometer mass as possible has to be used to decrease the mass loading effect.

The resonance frequencies of the plate measured with a accelerometer are lower than those of measured without accelerometer. Modal analysis is done by varying the mass of accelerometer. Mass loading effect of accelerometer reduces the natural frequencies to their lower values. The percentage of mass loading error is also decreases, as frequency is increased.

The experimental modal testing of a cantilever beam has been performed to obtain modal frequencies. These beams are excited by using Electrodynamics Shaker Excitation Technique. Modal frequencies are obtained by looking at peaks of transfer function Vs frequency graph obtained through sweep sine test. These experimental modal frequencies are found to comply with those obtained from finite element method.

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